Stochastic reduced order models for Bayesian estimation problems in fluid mechanics

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We are interested in Bayesian inverse problem for fluid dynamics in real-time application context. 3D unsteady and turbulent fluid systems encompass poor Kolmogorov N-width decays. Therefore, drastic dimensional reduction – typically from 10^7 to 10 – yields important truncation errors. Furthermore, for long-time integrations, these errors grow possibly without bound. Closures alleviate truncation errors but without preventing divergences in long-time extrapolations. Besides, for Bayesian inverse problems such as ensemble-based data assimilation (DA), a single reduced order model (ROM) solution is not enough. We must generate relevant solution priors instead.
We address this priors emulation problem with an energy-preserving stochastic closure called "Location uncertainty models" (LUM) [3, 4] and new statistical estimators based on stochastic calculus, signal processing and physics [4]. The deterministic ROM coefficients are obtained by a Galerkin projection whereas the correlations of the noises are estimated from the residual velocity, the physical model structure, and the evolution of the resolved modes. Posterior distributions are then easily computed assimilating measurement streams with a particle filter [2].
Whether we consider the priors [4] or the posteriors [2] of the ROM solution, our method greatly exceeds the state of the art, for ROM degrees of freedom smaller than 10 and moderately turbulent 3D flows (Reynolds number up to 300).
Our methodology is now implemented on the OpenFOAM-based ROM library ITHACA-FV [5]. In order to address higher Reynolds numbers, we now consider the hyper-reduction (DEIM) [1] of non-polynomial terms appearing in turbulence models (e.g., large eddy simulation).

References

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