

Hybrid Projection Methods with Recycling for Inverse Problems

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Abstract: Iterative hybrid projection methods have proven to be very effective for solving large linear inverse problems due to their inherent regularizing properties as well as the added flexibility to select regularization parameters adaptively. In this work, we develop Golub-Kahan-based hybrid projection methods that can exploit compression and recycling techniques in order to solve a broad class of inverse problems where memory requirements or high computational cost may otherwise be prohibitive. For problems that have many unknown parameters and require many iterations, hybrid projection methods with recycling can be used to compress and recycle the solution basis vectors to reduce the number of solution basis vectors that must be stored, while obtaining a solution accuracy that is comparable to that of standard methods. If reorthogonalization is required, this may also reduce computational cost substantially. In other scenarios, such as streaming data problems or inverse problems with multiple datasets, hybrid projection methods with recycling can be used to efficiently integrate previously computed information for faster and better reconstruction. Additional benefits of the proposed methods are that various subspace selection and compression techniques can be incorporated, standard techniques for automatic regularization parameter selection can be used, and the methods can be applied multiple times in an iterative fashion. Theoretical results show that, under reasonable conditions, regularized solutions for our proposed recycling hybrid method remain close to regularized solutions for standard hybrid methods and reveal important connections among the resulting projection matrices. Numerical examples from image processing show the potential benefits of combining recycling with hybrid projection methods.