Matrix-free Transfer Function Prediction using Model Reduction and Machine Learning

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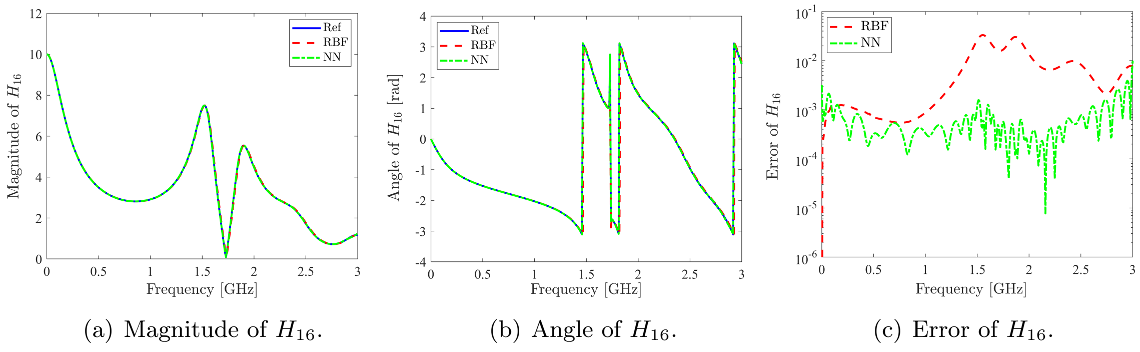
We propose a technique for fast predicting the transfer function of dynamical systems without information of system matrices by combining machine learning with model order reduction. The transfer function of a linear time invariant system can be written as $H(s) = CG(s)^{-1}B$. In some situations, it is difficult to obtain the individual system matrices that are implicitly included in $G(s)$. The only information available is the data of $G(s)$ at samples of frequency s. The proposed method derives a reduced-order model of the transfer function in the form of neural networks using limited data of $G(s)$. Discrete reduced transfer functions at training samples $s_i, i = 1, \ldots, l$, of the frequency are firstly generated based on the data of $G(s)$ at those training samples, i.e. $\hat{H}(s_i) = \hat{C}\hat{G}(s_i)^{-1}\hat{B}$. A reduced-order model of the original transfer function as a continuous function of the frequency is then learned using the data of the discrete reduced transfer functions. The original transfer function at any testing frequencies can then be quickly predicted using either a compact machine learning model or a deep learning model with a few layers. The discrete reduced transfer functions used as training data for machine learning, are guaranteed to be accurate thanks to a cheap and sharp error estimator. If only the data of the original transfer function are available, then the proposed machine/deep learning methods can be directly applied without generating the data of the discrete reduced transfer function. The proposed methods are tested on two PEEC models with many delays, efficiency and accuracy of the reduced-order models are demonstrated.

In the following Figure 1, we show some results for one of the two test examples used in the full paper, a powerbus model. In the figure, "Ref" refers to the original transfer function at 400 testing frequencies. The subscript "16" means the transfer function associated with the input port 6 and output port 1. "RBF" is the transfer function predicted by radial basis function (RBF) networks. "NN" corresponds to the transfer function predicted by deep feed forward neural networks. All the neural networks are trained using 200 training frequencies. It is seen that the neural networks predict the original transfer function accurately. Results for other input ports and output ports are also sufficiently accurate and are with errors below $O(10^{-2})$. The proposed methods have similar performance on the second example.



(a) Magnitude of H_{16}. (b) Angle of H_{16}. (c) Error of H_{16}.

Figure 1: Experimental results for a powerbus example.