Data driven reduced modelling of the Vlasov-Poisson equation

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Nowadays, deep learning neural networks are widely deployed and they flourish in many fields of applications. We propose to use them to develop a new Model Order Reduction (MOR) technique for kinetics models. It will be elaborated on the Vlasov-Poisson model. Indeed, this model describes the evolution of a charged particle distribution submitted to an electromagnetic field. The latter may be self-consistent, i.e. generated by the above-mentioned distribution. In consequence dynamics can be strongly non-linear.

There exists symplectic POD (Proper Orthogonal Decomposition) methods for parametric MOR in the self-consistent case [2]. It achieves to preserve the Hamiltonian structure of the model. Nonetheless, this approach seems insufficient in strongly non-linear cases. We propose a new, efficient and scalable process based on neural networks to reduce the ODE derived from Vlasov and learn a reduced model with a preserved Hamiltonian structure. We use Hamiltonian neural networks [4] conjointly with autoencoders neural networks [1]. The latter is being used for MOR as in [3] however we rely on coupled learning for both networks and well-chosen learning constraints to ensure an adequate reduction with good stability properties.

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