Data-Driven Model Reduction for Gas Network Digital Twins

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Spanning across and beyond the European continent, the gas transport network is a large-scale industrial application with critical relevance not only for current daily supply of natural gas, but also for the green energy transition namely through biogas and hydrogen. Therefore, dynamic gas network simulations are increasingly important for an interconnected smart energy grid, to compensate renewable energy production shortfalls and surpluses on short notice, since gas can be converted promptly into power, and excess renewable power can be converted back into gas.
Gas network models are typically based on the Euler equations for pipes, which lead to a system of nonlinear, parametric and hyperbolic partial differential-algebraic equations for pipeline networks. Using a port-Hamiltonian formulation along-side an analytic index reduction and spatial discretization yields a state-space input-output ordinary differential equation system:

$$\overbrace{\begin{pmatrix} E_p(\theta) & 0 \\ 0 & E_q \end{pmatrix}}^{E(\theta)} \overbrace{\begin{pmatrix} \dot{p} \\ \dot{q} \end{pmatrix}}^{\dot{x}} = \overbrace{\begin{pmatrix} 0 & A_{pq} \\ A_{qp} & 0 \end{pmatrix}}^{A} \overbrace{\begin{pmatrix} p \\ q \end{pmatrix}}^{x} + \overbrace{\begin{pmatrix} 0 & B_{pd} \\ B_{qs} & 0 \end{pmatrix}}^{B} \overbrace{\begin{pmatrix} s_p \\ d_q \end{pmatrix}}^{u} + \overbrace{\begin{pmatrix} 0 \\ F_c \end{pmatrix} + \begin{pmatrix} 0 \\ f_q(p,q,\theta) \end{pmatrix}}^{f(x,\theta)}$$

$$\underbrace{\begin{pmatrix} s_q \\ d_p \end{pmatrix}}_{y} = \underbrace{\begin{pmatrix} 0 & C_{sq} \\ C_{dp} & 0 \end{pmatrix}}_{C} \underbrace{\begin{pmatrix} p \\ q \end{pmatrix}}_{x}.$$

Due to the large-scale nature and manifold complexities of realistic gas network models, as well as the many-query setting of the practical application, model reduction can be used to reduce the order of the system, accelerating simulations, and hence, enable more scenarios to be evaluated prior to actual dispatch of denominations. Yet, the amalgamation of challenging features in gas transmission models makes the selection of suitable model reduction methods inherently difficult.
Given the model's hyperbolicity, non-smooth nonlinearity and parameter-dependence, we adapt, apply and compare data-driven system-theoretic model reduction methods [1], discuss model properties and algorithmic aspects affecting the reduced order system, and lay out extensibility to water networks, district heating networks and power networks. Furthermore, heuristic comparability of reduced order models is discussed [2].
Computationally, our open-source "morgen" (Model Order Reduction for Gas and Energy Networks) research software [3], a modular transient gas network simulation and model reduction prototyping stack, tests, compares and benchmarks model-solver-reductor ensembles for gas network digital twin development and model reduction research, which we demonstrate in numerical experiments.

References

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