Model reduction of convection-dominated partial differential equations via optimization-based implicit feature tracking

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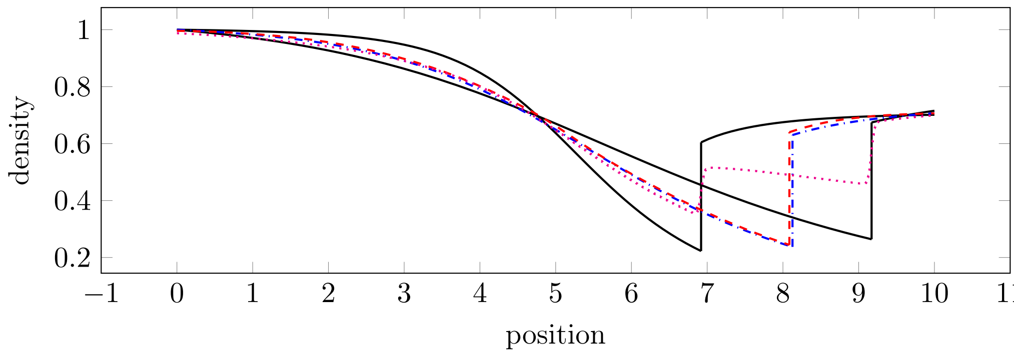
Partial differential equations (PDEs) that model convection-dominated phenomena often arise in engineering practice and scientific applications, ranging from the study of high-speed, turbulent flow over vehicles to wave propagation through solid media. The solutions of these equations are characterized by local features or disturbances that propagate throughout the domain as time evolves or a system parameter varies. Numerical methods to approximate these solutions require stabilization and fine, usually adaptive, grids to adequately resolve the local features, which lead to expensive discretizations with a large number of degrees of freedom. Projection-based model reduction methods tend to be ineffective in reducing the computational cost of such problems due to a slowly decaying Kolmogorov n-width of the solution manifold.

To avoid the fundamental linear reducibility limitation associated with convection-dominated problems, we construct a nonlinear approximation by composing a low-dimensional linear space with a parametrized domain mapping [1, 2]. The linear space is constructed using the method of snapshots and POD; prior to compression, each snapshot is composed with a mapping that causes its local features to align (same spatial location) with the corresponding features in all other snapshots. The parametrized domain mapping is chosen such that the local features present in the linear space deform to the corresponding features in the solution being approximated, effectively removing the convection-dominated nature of the problem. The domain mapping is determined implicitly through the solution of a residual minimization problem, rather than relying on explicit sensing/detection. We provide numerous numerical experiments to demonstrate the effectivity of the proposed method on benchmark problems from computational fluid dynamics.



Comparison of the HDM, L^2 projection (without snapshot alignment), and ROM-IFT on a nozzle flow problem with two training parameters. Legend: training snapshots (——), HDM solution at test parameter (-·-·-), L^2 projection of test parameter onto reduced basis (······), and ROM-IFT solution at test parameter (- - -).

References

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[2] T. Taddei. A registration method for model order reduction: data compression and geometry reduction. *SIAM Journal on Scientific Computing*, 42(2):A997–A1027, 2020.