Non-intrusive adaptive surrogate modeling of parametric frequency-response problems

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Numerical methods for time-harmonic wave propagation phenomena are often computationally intensive, especially in mid- and high-frequency regimes, making a direct frequency response analysis prohibitively expensive. In this framework, starting from few expensive solves of the problem, model order reduction (MOR) methods can provide a reliable and cheap-to-evaluate approximation of the frequency response of the system. In this talk, we describe an MOR approach for parametric frequency-response problems, where the high-fidelity problem

$$\begin{cases} zE(\mathbf{p})\mathbf{x}(z,\mathbf{p}) = A(\mathbf{p})\mathbf{x}(z,\mathbf{p}) + \mathbf{b}(z,\mathbf{p}) \\ \mathbf{y}(z,\mathbf{p}) = C(\mathbf{p})\mathbf{x}(z,\mathbf{p}) + \mathbf{d}(z,\mathbf{p}) \end{cases}$$

models not only the impact of the frequency z on the system response, but also that of additional design and/or uncertain parameters $\mathbf{p}$. Our approach is non-intrusive, i.e., we do not require access to the matrices/operators defining the underlying high-fidelity problem.
Our proposed method relies on *minimal rational interpolation* [2] for the surrogate modeling of the frequency dependence, for few fixed values of the parameters $\mathbf{p} \in \{\mathbf{p}_j\}_{j=1}^T$. In this step, only the dependence on z is taken into account, and the T resulting surrogates $\{\widetilde{\mathbf{y}}_j\}_{j=1}^T$ (with $\widetilde{\mathbf{y}}_j(z) \approx \mathbf{y}(z,\mathbf{p}_j)$ for all j) are rational functions of z. We describe how an adaptive selection of the frequency sample points can be carried out even in a non-intrusive framework.
Then, the T different z-surrogates, properly converted to a modal form

$$\left\{ \widetilde{\mathbf{y}}_j \ : \ z \ \mapsto \ \sum_i \frac{r_i^j}{z-\lambda_i^j} \right\}_{j=1}^T,$$

are combined to obtain a global approximation with respect to both frequency and parameters [1]. We also describe how locally adaptive sparse grids can be applied over $\mathbf{p}$-space to (i) weaken the curse of dimension and (ii) perform non-intrusively an adaptive sampling of the parameters.
Numerical examples in modeling of electrical circuits and elasto-dynamic systems will also be presented, providing evidence of the approximation quality and computational efficiency of the surrogate models obtained with the proposed technique.

References

[1] F. Nobile and D. Pradovera. Non-intrusive double-greedy parametric model reduction by interpolation of frequency-domain rational surrogates. *ESAIM: Mathematical Modelling and Numerical Analysis*, 55(5):1895–1920, 2021.

[2] D. Pradovera. Interpolatory rational model order reduction of parametric problems lacking uniform inf-sup stability. *SIAM Journal on Numerical Analysis*, 58(4):2265–2293, 2020.