Data-Driven Linearization of Nonlinear Finite Element Analyses

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Nonlinear finite element analysis is gaining attention for the modeling of complex mechanical assets. The nonlinear nature of these systems is another issue that complicates their analysis. This can be seen, e.g., for system identification and Kalman filtering. In this talk, we propose a method that approximates the output of a nonlinear dynamical system by a linear one. Moreover, the second order nature of the nonlinear system is conserved by the linear approximation.

The Full Order Models (FOMs) we consider should be treated as a black box, since knowledge of the underlying dynamical system may not be available. Instead of looking for a linearization of the whole FOM, we focus on the response of the system at a specific input. Given the input signal $u(t)$, we therefore look for an approximation of the system's response $y_u(t)$. To this end we define a new data-driven system-theoretic method, which we call tLS-AAA. This method is able to find a linear Reduced Order Model (ROM) whose impulse response accurately approximates $y_u(t)$.

The base of tLS-AAA is the AAA algorithm [2], and in particular its Least Squares form introduced in [1]. This form is useful to delete possible unstable poles obtained in the AAA procedure. The ROM generated by tLS-AAA is the linear second order system

$$\Sigma_r : \qquad \begin{cases} M_r\ddot{\boldsymbol{x}}_r(t) + E_r\dot{\boldsymbol{x}}_r(t) + A_r\boldsymbol{x}_r(t) = B_r\delta(t) \\ y_r(t) = \varepsilon_0\delta(t) + \varepsilon_1 + \boldsymbol{\alpha}^T\boldsymbol{x}_r(t) + \boldsymbol{\beta}^T\dot{\boldsymbol{x}}_r(t) \end{cases}, \tag{1}$$

where the coefficients ε_0, ε_1, $\boldsymbol{\alpha}$, and $\boldsymbol{\beta}$ can be found by solving the LS problem

$$\min \|y_u - y_r\|_2. \tag{2}$$

Since this method focuses on the time domain, we call it Time LS-AAA (tLS-AAA). Even though the ROM (1) is generated for the specific input $u(t)$, we show that it can easily deal with a larger set of input signals. We consider different test cases, and we demonstrate the accuracy and flexibility of this method.

References

[1] S. Costa et al. AAA-least squares rational approximation and solution of Laplace problems. *arXiv preprint arXiv:2107.01574*, 2021.

[2] Y. Nakatsukasa et al. The AAA algorithm for rational approximation. *SIAM Journal on Scientific Computing*, 40:A1494–A1522, 2018.