

Adaptive POD-DEIM correction for Turing pattern approximation in reaction-diffusion PDE systems

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In this talk we focus on the stabilization of the POD-DEIM technique for the numerical approximation of Turing patterns, that are stationary solutions of reaction-diffusion PDE (RD-PDE) systems. We show that solutions of surrogate models built by classical POD-DEIM (see e.g. [2, 3]) exhibit an unstable error behaviour over the dimension of the reduced space. To overcome this drawback, we add a correction term as for e.g. [5, 4] that provides missing information to the reduced model and we apply the POD-DEIM technique to the corrected model (see [1]). To further improve the computational efficiency, we propose an adaptive version of this algorithm in time that accounts for the peculiar dynamics of the RD-PDE in presence of Turing instability. We show the effectiveness of the proposed methods in terms of accuracy and computational cost for a selection of RD systems, i.e. FitzHugh-Nagumo, Schnackenberg and the morphochemical DIB models, with increasing degree of nonlinearity and more structured patterns.

References

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