Neural networks investigation of bifurcating phenomena in fluid-dynamics

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Bifurcating phenomena, i.e. sudden changes in the qualitative behavior of the system linked to the non-uniqueness of the solution naturally arise in several fields. Since the reconstruction of bifurcation diagrams requires a many-query context, which is usually unaffordable using high fidelity simulations, we propose a combination of Reduced Order Models (ROMs) and Machine Learning techniques to reduce the computational burden associated with the investigation of such complex phenomena.

This work aims to show the applicability of the Reduced Basis (RB) model reduction and Artificial Neural Network (ANN), utilizing the POD-NN approach and its physics-informed variant [2, 1], to analyze multi-parameter bifurcating applications in fluid-dynamics.

We considered the Navier-Stokes equations for a viscous, steady, and incompressible flow: (i) in a planar straight channel with a narrow inlet of varying width and (ii) in a triangular parametrized lid-driven cavity. Within this context, we present a new empirical strategy to employ the RB and ANN coefficients for a non-intrusive detection of the bifurcation points [3].

Finally, we introduce a newly developed ROM methodology based on Graph Neural Network, with powerful applications to general parametrized PDEs and branches classification when dealing with bifurcating phenomena [4].

References

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