Conditional gradient-based Identification of Non-linear Dynamics

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Sparsity of nonlinear dynamical models extracted from data is an attractive feature for the numerical efficiency of surrogate models as well as for the inference of interpretable governing laws from observations. Symbolic regression approaches like *Sparse Identification of Nonlinear Dynamics* (SINDy) [1] solve a data-fitting problem such as

$$\min_{\xi \in \mathbb{R}^n} \sum_{i=1}^{k} \left\| \dot{x}_i - \sum_{j=1}^{n} \xi_j f_j(x_i) \right\|^2$$

for coefficients ξ_j for a potentially huge library of n atoms f_j, subject to some sparsity-enhancing constraint or penalization such as the *Least Absolute Shrinkage and Selection Operator* [4] leading to a ℓ^1 constraint $\|\xi\|_1 \le \alpha$.

A large variety of methods are available for solving the resulting optimization problem. We propose the efficient first-order Conditional Gradient [3] algorithm CINDy (named as an hommage to SINDy) for its solution [2]. In comparison to the most prominent alternative algorithms, the new algorithm shows significantly improved performance on several essential issues like sparsity induction, structure preservation, noise robustness, sample efficiency, and predictive power. We demonstrate these advantages on several dynamics from the field of synchronization (Fig. 1), particle dynamics, and enzyme chemistry.

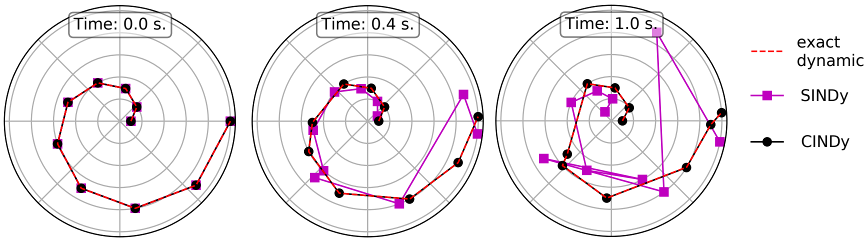

Figure 1: Trajectory comparison of a Kuamoto model identified with SINDy and CINDy.

References

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