Adapting Reduced Models for Importance Sampling

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We want to compute the probability of a rare event of the form $P(S(X) > L)$, with X a random vector in $\mathbf{R^d}$, with a computable density (up to a normalization constant), and S a real function that is very expensive to compute. We assume that we have a budget, i.e., a fixed number of complete evaluations of S, and that we can also build reduced-order models of S, which can be iteratively refined when new values of S are computed. We propose a fully adaptive algorithm to iteratively build an importance sampling distribution and draw from it, the objective being the evaluation of the probability of the rare event. The importance distribution takes the form of a Gibbs measure based on the current reduced order model, with parameters adjusted to minimize the relative entropy with respect to the target rare event probability distribution. A sequential Monte-Carlo technique generates from this Gibbs measure a swarm of particles, which is used as an empirical approximation of the importance distribution. At each iteration, a sample is drawn from the current empirical measure, the exact value of S calculated, the estimate of the rare event updated, as well as the reduced order model, the Gibbs measure, and the empirical sampling distribution. After the detailed presentation of the algorithm, we will give some heuristics. Some numerical results will also illustrate its relevance.