Data-Driven Enhanced Model Reduction for Bifurcating Models in Computational Fluid Dynamics

M. W. Hess[1], A. Quaini[2], and G. Rozza[1]

[1]*SISSA mathLab, International School for Advanced Studies, via Bonomea 265, I-34136 Trieste, Italy,* mhess@sissa.it, grozza@sissa.it
[2]*Department of Mathematics, University of Houston, Houston, Texas, 77204, USA,* quaini@math.uh.edu

We investigate various data-driven methods to enhance projection-based model reduction techniques with the aim of capturing bifurcating solutions. In particular, we use a three-step procedure of proper orthogonal decomposition (POD), dynamic mode decomposition (DMD) and manifold interpolation [7] to compute parametrized reduced order models for a cavity undergoing Hopf bifurcations with varying Grashof number [3], [6]. The approach can be extended to solution trajectories, which are not necessarily on the limit cycle but start from a common initial state for all trajectories correponding to the considered parameters with the Hankel-DMD [1], [5]. Addtionally, it is possible to resolve multiple solutions for a channel flow undergoing a pitchfork bifurcation [2] using localized model reduction and artificial neural networks [4], [5].

References

[1] H. Arbabi and I. Mezic. Ergodic theory, dynamic mode decomposition, and computation of spectral properties of the koopman operator. *SIAM Journal on Applied Dynamical Systems*, 16(4):2096–2126, 2017.

[2] D. Drikakis. Bifurcation phenomena in incompressible sudden expansion flows. *Physics of Fluids*, (9):76–87, 1997.

[3] A. Gelfgat, P. Bar-Yoseph, and A. Yarin. Stability of multiple steady states of convection in laterally heated cavities. *Journal of Fluid Mechanics*, 388:315–334, 1999.

[4] M. W. Hess, A. Quaini, and G. Rozza. A comparison of reduced-order modeling approaches using artificial neural networks for PDEs with bifurcating solutions. *ETNA - Electronic Transactions on Numerical Analysis*, 56:52–65, 2022.

[5] M. W. Hess, A. Quaini, and G. Rozza. Data-driven enhanced model reduction for bifurcating models in computational fluid dynamics. https://arxiv.org/abs/2202.09250, 2022.

[6] M. W. Hess, A. Quaini, and G. Rozza. A data-driven surrogate modeling approach for time-dependent incompressible navier-stokes equations with dynamic mode decomposition and manifold interpolation. http://arxiv.org/abs/2201.10872, 2022.

[7] R. Zimmermann. Manifold interpolation. In P. Benner, S. Grivet-Talocia, A. Quarteroni, G. Rozza, W. Schilders, and L. M. Silveira, editors, *Model Order Reduction, Volume 1: System- and Data-Driven Methods and Algorithms*, pages 229–274. De Gruyter, 2021.