Generating reduced order models parallel in time via random sampling

J. Schleuß[1], K. Smetana[2], and L. ter Maat[3]

[1]*Faculty of Mathematics and Computer Science, University of Münster, Einsteinstraße 62, 48149 Münster, Germany (julia.schleuss@uni-muenster.de)*
[2]*Department of Mathematical Sciences, Stevens Institute of Technology, 1 Castle Point Terrace, Hoboken, NJ 07030, United States of America (ksmetana@stevens.edu)*
[3]*Department of Applied Mathematics, University of Twente, P.O. Box 217, 7500 AE Enschede, The Netherlands*

In this talk, we generate reduced basis functions defined in space that can be combined with time stepping schemes within model order reduction methods. We propose, to the best of our knowledge, for the first time an embarrassingly parallel reduced basis construction in time [5]. Moreover, we especially target time-dependent partial differential equations (PDEs) with coefficients that are arbitrarily rough in both space and time.
In detail, we perform several simulations of the PDE for few time steps in parallel, starting at different, randomly drawn start points, prescribing random initial conditions. Applying a singular value decomposition to a subset of the so obtained snapshots yields the reduced basis. This facilitates constructing the reduced basis functions parallel in time. To select start time points for the temporally local PDE simulations, we suggest using a data-dependent probability distribution. To this end, we represent the time-dependent data functions of the PDE as matrices, where each column of a matrix corresponds to one time point in the grid of the time discretization. Subsequently, we employ column subset selection techniques from randomized numerical linear algebra [3] such as leverage score sampling.
Each local in time simulation of the PDE with random initial conditions approximates a local approximation space in one time point that is optimal in the sense of Kolmogorov (cf., e.g., [1, 4]). These optimal local approximation spaces are spanned by the left singular vectors of a compact transfer operator that maps arbitrary initial conditions to the solution of the PDE in a later point of time. By solving the PDE locally in time with random initial conditions, we construct local ansatz spaces in time that converge provably at a quasi-optimal rate and allow for local error control (cf. [2]).
Numerical experiments demonstrate that the proposed method can outperform existing methods like the proper orthogonal decomposition even in a sequential setting.

References

[1] I. Babuška and R. Lipton. Optimal local approximation spaces for generalized finite element methods with application to multiscale problems. *Multiscale Model. Simul.*, 9(1):373–406, 2011.

[2] A. Buhr and K. Smetana. Randomized Local Model Order Reduction. *SIAM J. Sci. Comput.*, 40(4):A2120–A2151, 2018.

[3] P. Drineas and M. W. Mahoney. RandNLA: Randomized Numerical Linear Algebra. *Commun. ACM*, 59(6):80–90, 2016.

[4] J. Schleuß and K. Smetana. Optimal local approximation spaces for parabolic problems. *Multiscale Model. Simul., to appear*, 2022+.

[5] J. Schleuß, K. Smetana, and L. ter Maat. Randomized quasi-optimal local approximation spaces in time. *https://arxiv.org/abs/2203.06276*, 2022.