A greedy MOR method for the tracking of eigensolutions to parametric PDEs

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The present talk concerns the numerical solution to eigenvalue problems arising from parametric partial differential equations. Such mathematical models are used to describe various phenomena in engineering and design, statistics, biology and life science. While uncertainty quantification techniques for partial differential equations with parametric/stochastic inputs is an active research field, less attention has been devoted to eigenvalue problems.
Before any reasonable discretization, the regularity of the eigenvalue/eigenfunction solutions with respect to the parameters should be discussed. From this perspective, the main challenge is represented by the eigenvalue/eigenfunction crossings that might happen as the parameter values change. It is then of paramount importance to properly track all correct modes. So far, only few investigations have been carried on, mainly related to isolated eigenvalues [1, 2].
We consider an elliptic eigenvalue problem with a multi parametric dependence and we design an adaptive algorithm which minimizes the number of solves to guarantee the correct matching of the eigenvalues within a prescribed tolerance. The proposed scheme can be interpreted as a greedy model order reduction (MOR) method, where the greedy selection of the parameter values is guided by the a-priori cost functional introduced in [3] combined with a suitable a-posteriori strategy, capable of detecting possible mistakes in the output of the a-priori phase.
Several numerical examples are presented to display the performances and the effectiveness of the method.

References

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