Stability Analysis of Reduced Basis Model Predictive Control for Parametrized Optimal Control Problems

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Model Predictive Control (MPC) is a well established approach to solve infinite horizon optimal control problems. Since optimization over an infinite time horizon is, in general, infeasible, the method determines a suboptimal feedback control by repeatedly solving finite time optimal control problems. Although MPC has been successfully used in many applications, applying MPC to large-scale systems – arising, e.g., through discretization of partial differential equations – requires the solution of high-dimensional optimal control problems and thus poses immense computational effort.

We consider systems governed by parametrized parabolic partial differential equations and employ the reduced basis method (RB) as a low-dimensional surrogate model for the finite time optimal control problem. The reduced order optimal control serves as the feedback control for the MPC of the original large-scale system. We analyze the proposed RB-MPC approach by first developing rigorous *a posteriori* error bounds for the errors in the optimal control and the associated cost functional. These bounds can be evaluated efficiently in an offline-online computational procedure and therefore allow us to guarantee asymptotic stability of the closed-loop system using the RB-MPC approach. Furthermore, we propose an adaptive strategy based on the error bounds to choose the optimal horizon length of the finite time optimal control problem. We present numerical results to validate our approach.

Although we can provide rigorous results only for linear-quadratic problems, our approach may also provide a guideline for nonlinear problems. To this end, we compare two methods: First, we consider a (simple) piecewise linearization approach that allows to invoke the theory on linear-quadratic problems. In the second approach, we consider the nonlinear problem and employ the Empirical Interpolation Method (EIM) to efficiently solve the associated optimal control problem. A semilinear parabolic model problem is considered in order to discuss and compare the two proposed approaches.