Reduced Basis Methods for Time-Harmonic Maxwell's Equations

Anna Sanfilippo[1], **Francesco Ballarin**[2], **and Ana María Alonso Rodríguez**[1]

[1] *University of Trento, Trento, Italy*
[2] *Catholic University of the Sacred Heart, Brescia, Italy*

One of the main practical difficulties concerning the solution of parametrized problems derived from Maxwell's equations is the required running time. These problems often use edge, or Nédélec, finite elements for the approximation of the main unknown that belongs to the space $H(\mathrm{curl}, \Omega)$ properly modified according to the boundary conditions. The aim of this work is to study the performance of Reduced Basis (RB) methods when the first family of Nédélec finite elements, see [2], is used. For all the tests we employ the RBniCSx software [1], that is the new version of RBniCS, based on FEniCSx where complex numbers are available. Thus, our studies are focused on applying Reduced Basis (RB) methods to edge finite elements and verifying their effectiveness in the RBniCSx environment, since in our knowledge it is the first time that edge elements are use in a RBniCS software. In particular, we use both Proper Orthogonal Decomposition (POD) and greedy sampling as strategy to generate the RB space.

We use time harmonic Maxwell's equations with impedance boundary conditions parametrized in terms of the frequency ω, see [3], i.e.

$$\begin{aligned} \nabla \times (\mu^{-1} \nabla \times \boldsymbol{E}) - (\omega^2 \varepsilon + i\omega\sigma)\boldsymbol{E} = \boldsymbol{F} \quad & \text{in} \quad \Omega, \\ \boldsymbol{n} \times \boldsymbol{E} = \boldsymbol{0} \quad & \text{on} \quad \Gamma_{int}, \\ (\mu^{-1} \nabla \times \boldsymbol{E}) \times \boldsymbol{n} - i\omega\lambda(\varepsilon_0 \mu_0^{-1})^{1/2} \boldsymbol{E}_T = \boldsymbol{g} \quad & \text{on} \quad \Gamma_{out}, \end{aligned} \tag{1}$$

where $\boldsymbol{E}_T := (\boldsymbol{n} \times \boldsymbol{E}|_{\Gamma_{out}}) \times \boldsymbol{n}$ and the domain Ω is a cube with a cubical cavity, i.e. $\Omega := (-1, 1)^3 \setminus (-1/2, 1/2)^3$.

For this problem it can be proved that the sesquilinear form is coercive on the Hilbert space $X := \{\boldsymbol{u} \in H(\mathrm{curl}, \Omega) \mid \boldsymbol{n} \times \boldsymbol{u} = \boldsymbol{0} \text{ on } \Gamma_{int};\ \boldsymbol{u}_T \in L^2(\Gamma_{out}, \mathbb{C}^3) \text{ on } \Gamma_{out}\}$. Thus, the complex valued Lax-Milgram lemma holds and it is possible to determine explicitly the *a-posteriori* error bound for the greedy procedure through the coercivity constant, rather than with a lower bound of the the inf-sup constant as done in [2]. The numerical results have attained the theoretical expectations and we observed that the speedup numbers for time harmonic Maxwell's equations are very significant.

References

[1] RBniCSx project. `https://github.com/RBniCS/RBniCSx`.

[2] M. W. Hess and P. Benner. Fast Evaluations of Time-Harmonic Maxwell's Equations using the Reduced Basis Method. *IEEE Transactions on Microwave Theory and Techniques*, 61:2265–2274, 2013.

[3] K. Kirchner, K. Urban, and O. Zeeb. Maxwell's Equations for Conductors with Impedance Boundary Conditions: Discontinuous Galerkin and Reduced Basis Methods. *ESAIM: M2AN*, 50.6:1763–1787, 2016.