A structure-preserving DEIM formulation for non-linearly stable hROMs of the incompressible Navier-Stokes equations

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An issue of increasing interest in projection-based reduced order modeling of conservation laws is the preservation of the conservative structure underlying such equations at the reduced level ([2], [3]). A non-linearly stable POD-Galerkin ROM of the incompressible Navier-Stokes equations that globally conserves kinetic energy (in the inviscid limit), momentum and mass on periodic domains was constructed in [2]. The quadratic nonlinearity in the convection operator was dealt with using an exact tensor decomposition to eliminate the dependence of the computational scaling of the ROM on the FOM dimensions. However, such a cubic tensor decomposition is not always feasible: in case many POD modes are required (cases with slow Kolmogorov N-width decay), the exact decomposition becomes prohibitively expensive. One possible solution is the use of hyper-reduction methods such as the discrete empirical interpolation method (DEIM) [1]. The DEIM generally does not retain the conservative structure of the ROM to which it is applied and, as a consequence, non-linear stability of the ROM of the incompressible Navier-Stokes equations proposed in [2] is no longer guaranteed.

In this work we propose a novel DEIM formulation that allows us to construct a non-linearly stable hyper-reduced order model (hROM) of the incompressible Navier-Stokes equations. The hROM has the same mass, momentum and energy conservation properties as the ROM proposed in [2], but does not suffer of prohibitive computational scaling when the number of POD modes is increased. In detail, the proposed structure-preserving DEIM formulation is a rank-one correction of the conventional DEIM method which can be calculated efficiently with the Sherman-Morrison inverse formula. This results in a structure-preserving DEIM formulation that has equivalent computational scaling as the conventional DEIM, but provides provably stable, structure-preserving hROMs.

We implement the structure-preserving DEIM formulation in a hROM of the incompressible Navier-Stokes equations based on the ROM proposed in [2]. The computational cost associated with the hROM is compared to the ROM employing the cubic tensor decomposition for several convection-dominated flow configurations with slow Kolmogorov N-width decay, for example: turbulent channel flow, shear-layer instabilities, and the Taylor-Green vortex.

References

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