Time extrapolation technique applied to POD-based ROM

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Reduced-order models (ROMs) based on the proper orthogonal decomposition (POD, [3]) are frequently used to solve partial differential equations more efficiently. The ROM must be trained off-line following the snapshots method [2]: a set of solutions is computed up to the training time, t_{train}, by means of the full-order model (FOM), the name given to the discretization of the equation of interest. POD-based ROMs are not able to compute solutions of advection-dominated problems beyond t_{train}. In this work, a novel method based on a coordinate transformation [1], called CT-ROM, is presented to allow for the extrapolation of solutions beyond the training time.
Figure 1 shows the difference between the standard ROM (left) and the CT-ROM (right) applied to the linear advection-diffusion equation $\partial_t u + a\partial_x = \nu\partial_{xx}u$, with $a = 0.5$ and $\nu = 0.001$. The initial Gaussian profile, the computed solution at t_{train} and $T > t_{\text{train}}$ are plotted together in a case where the Peclet number is $Pe = 5$.

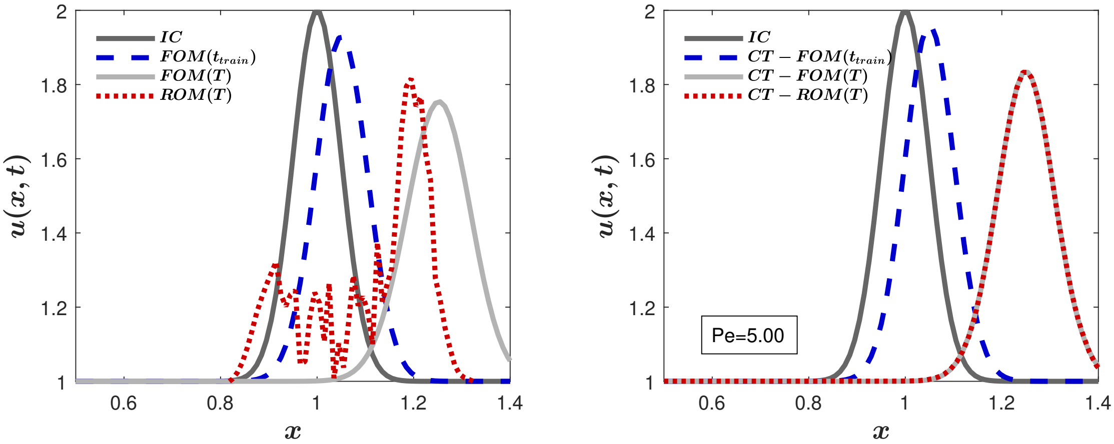

Figure 1: Solutions computed with the FOM/ROM (left) and with the CT-FOM/CT-ROM (right).

The CT-ROM strategy shows promising results in different scenarios, such as 1D linear advective equation with diffusion source term. This method has been applied to more examples: linear advective equation with reaction source term, systems of coupled linear equations and the non-linear inviscid Burgers' equation [3].

References

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