Nonlinear Model Order Reduction for Three-dimensional Discretized FE Models using Graph Convolutional Autoencoders

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Besides classical data-driven dimensionality reduction techniques like proper orthogonal decomposition (POD), autoencoders (AEs) have been established as a nonlinear alternative in the field of model order reduction (MOR) as in [1]. They are well-suited to find low-dimensional representations of high-dimensional systems by incorporating a "bottleneck" in their architecture. Compared to classical approaches, AEs relying on fully connected layers have disproportionately more adjustable parameters resulting in an expensive offline trainings-phase that requires vast amounts of resources.
An often used remedy are convolutional neural networks (CNNs) [2, 3]. They can exploit geometric patterns in the data and construct complex patterns from small and basic patterns that are stored in their filters. Often, they are used in Euclidean domains with grid-based structured data. Unfortunately, data obtained from finite element (FE) models are usually not spatially discretized in a grid-like structure but in a complex three-dimensional shape. Hence, the generalization of CNNs to such models is not trivial.
FE models, however, can be represented in a graph-like structure enabling the usage of graph convolutions. They share with the conventional version the property of recognizing local patterns but lack the feature of dimensionality reduction. Hence, graph convolution autoencoders (GCA) must be combined with mesh reduction techniques to be successfully applied in the context of MOR similar as it already has been done in computer vision [4].
This enables the efficient identification of low-dimensional coordinates with which the system can be described. With these coordinates, the connection between simulation parameters and system behavior can be learned efficiently within the autoencoder itself or with any other regression algorithm.
We demonstrate the capabilities of this approach on high-dimensional static as well as dynamic problems, including human body models and crash simulations [5]. Furthermore, the methodology is compared to more widespread approaches like POD-based data-driven model reduction.

References

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