Two-Scale Reduction of LOD Multiscale Models

T. Keil[1] **and S. Rave**[1]

[1] *University of Münster, Germany*

The Localized Orthogonal Decomposition (LOD) method is a successful recent approach for the numerical solution of multiscale PDE problems with or without scale separation (see [3] and references therein). The LOD is based on the idea of splitting the solution space into a negligible fine-scale space, given by the kernel of a coarse-mesh interpolation operator, and an energy-orthogonal multiscale space, in which the solution is sought. To obtain the multiscale space, fine-scale corrector problems need to be solved for each basis function of the coarse-mesh finite element space. Due to the fast decay of the corrector functions, the corrector problems can be approximated by restricting the domain to a patch of coarse-elements around the support of the respective basis function. Thus, similar to other numerical multiscale methods, the computational effort is split into the solution of many small local fine-scale problems and the solution of a globally coupled effective coarse-scale problem.

To accelerate the solution of large parameterized multiscale problems in a multi-query context, a natural approach is to combine the LOD with Reduced Basis (RB) methods. In [1] an RB-LOD scheme was introduced in which the corrector problems are replaced by corresponding RB surrogate models. However, for very large problems also the solution of the coarse-scale problem requires relevant work, which is not addressed by this approach. Also, the error in the LOD solution induced by the RB approximation of the corrector problems is not rigorously controlled.

In this contribution we present an efficient two-stage two-scale model reduction approach for the LOD which takes both the fine and coarse scale of the problem into account [2]. It is based on a new two-scale formulation of the LOD for which we show well-posedness and stability. By applying the RB method to this formulation, we obtain a surrogate model of small dimension independent of the size of the coarse- and fine-scale meshes. Rigorous a priori and a posteriori bounds control the model reduction error. Numerical experiments yield speedup factors of up to 1000 over an approach similar to [1] where only the corrector problems are reduced.

References

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