Dynamic Mode Decomposition for Continuous Port-Hamiltonian Systems

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We present a novel physics-informed system identification method to construct a passive linear time-invariant system. In more detail, for a given quadratic energy functional, measurements of the input, state, and output of a system in the time domain, we find a realization that approximates the data well while guaranteeing that the energy functional satisfies a dissipation inequality. One of the main advantages of requiring the learned model to satisfy a dissipation inequality is that whenever the model is coupled with another passive model via a powerconserving or dissipative interconnection, then the coupled model is also passive. To this end, we use the framework of port-Hamiltonian (pH) systems [5] and modify the dynamic mode decomposition (DMD) [3] to be feasible for continuous-time pH systems. We propose an iterative numerical method, to solve the corresponding least-squares minimization problem of the form

$$\min \left\| \mathcal{Z} - (\widetilde{\mathcal{J}} - \widetilde{\mathcal{R}})\mathcal{T} \right\|_{\mathrm{F}} \qquad \text{s. t. } \widetilde{\mathcal{J}} = -\widetilde{\mathcal{J}}^T \text{ and } \widetilde{\mathcal{R}} \succeq 0, \tag{1}$$

where $\mathcal{Z}$ and $\mathcal{T}$ correspond to the discrete-time data and $\widetilde{\mathcal{J}}$ and $\widetilde{\mathcal{R}}$ to the matrices of the continuous-time pH system. The proposed method divides the original problem into two subproblems, by alternately fixing $\widetilde{\mathcal{J}}$ respectively $\widetilde{\mathcal{R}}$ and optimizing solely over the remaining matrix. The resulting subproblems are a skew-symmetric and a symmetric positive semi-definite Procrustes problem. The skew-symmetric Procrustes problem can be solved analytically [1] and algorithmic solutions are available for the symmetric positive semi-definite Procrustes problem [2]. We present a modification of the proposed Fast Gradient Method, based on [4], with which it is possible to use the solution of the skew-symmetric Procrustes problem iteratively. For an effective initialization we analyze the least-squares problem in a weighted norm,

$$\min \left\| \mathcal{T}^T \mathcal{Z} - \mathcal{T}^T(\widetilde{\mathcal{J}} - \widetilde{\mathcal{R}})\mathcal{T} \right\|_{\mathrm{F}} \qquad \text{s.t. } \widetilde{\mathcal{J}} = -\widetilde{\mathcal{J}}^T \text{ and } \widetilde{\mathcal{R}} \succeq 0, \tag{2}$$

for which we present the analytical minimum-norm solution. The efficiency of the proposed method is demonstrated with several numerical examples.

References

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