Spectral approximation of Lyapunov operator equations with applications in high dimensional non-linear feedback control

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Solving the infinite horizon Hamilton-Jacobi-Bellman equation in high dimensions is a difficult task that appears in many applications. Instead of calculating the value function directly we present a new approach that will use a spectral decomposition of the solution to an operator Lyapunov equation with respect to a finite rank infinite-time admissible observation operator. Denoting by A the generator of the semigroup of the underlying dynamics and a bilinear form $\langle \cdot, \cdot \rangle_g$ representing the cost, we focus on the operator Lyapunov equation:

$$\langle A^* \phi, \psi \rangle_P + \langle \phi, A^* \psi \rangle_P + \langle \phi, \psi \rangle_g = 0 \qquad \forall \phi, \psi \in \mathcal{D}(A)$$

We will show that the solution P admits a spectral decomposition with rapidly decaying eigenvalues, which makes a finite rank approximation feasible. In the simple case of a linear quadratic problem this approach is equivalent to finding the best low-rank solution of an ordinary Lyapunov equation and will lead to linear eigenfunctions. Motivated by this, we then also reformulate the problem as an optimization problem over the manifold of all fixed rank operators. Two main benefits of this approach can be expected: For once, the representations in terms of a tensor train (TT), which we will use for discretization, have lower ranks. Secondly we will use Riemannian optimization to perform a simultaneous update for both the control and the value function. This will significantly speed up the computation. We show numerical results for simple, but non-linear, systems without control, which prove the concept viable.