

# Efficient Hyper-Reduction of contact problems treated by Lagrange multipliers.

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The resolution of contact mechanics problems is often time consuming because of the non-linearity of the contact constraints. For contact problems treated by Lagrange multipliers (ensuring the Signorini contact conditions to be respected), applying reduced-order modeling remains challenging due in particular to the non-negativity constraint on the Lagrange multipliers [1]. We focus here on the Hybrid Hyper-Reduction (HHR) of contact problems proposed in [3], based on a reduced integration domain (RID). Hence a reliable reduced dual basis is obtained by restricting the full order dual basis to the RID. The HHR model is then a saddle-point problem that must respect the necessary solvability condition associated to the well-posedness of the reduced problem. In the discrete setting, this condition imposes the projected contact rigidity matrix to be of full row rank. This may lead to an extension of the primal POD reduced basis [3]. By highlighting the strong link between the condition number of the projected contact rigidity matrix and the precision of the dual reduced solutions, we propose two enrichment methods aiming at respecting the solvability condition and controlling the size of the primal reduced basis with respect to the required precision on the dual solutions [4]. For large parametric variation of the contact zone, the reachable dual precision may remain limited due to the high nonlinearity of the dual solutions. We then propose a clustering strategy (derived from [2]) on the parametric space in order to circumvent this issue through piecewise low-rank approximations. On each cluster, a local accurate HHR model is built thanks to the enrichment strategies. The overall solutions is then deeply improved while preserving an interesting compression of both primal and dual bases.

## References

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