Inf-Sup-Constant-Free Error Estimation for Linear Parametric Systems

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Standard *a posteriori* state error estimation for model order reduction relies on the inf-sup constant [3]. The *a posteriori* error estimation for systems with very small or vanishing inf-sup constant poses a challenge, since the estimator is inversely proportional to the inf-sup constant, resulting in pessimistic error estimation. Such systems appear in electromagnetics where the inf-sup constant values are zero or close to zero, at or near resonant frequencies. In this work, we introduce two efficient *a posteriori* error estimators that are independent of the inf-sup constant.

The first error estimator [1] (called Est1 hereafter) is a state error estimator targeted towards general linear parametric systems. Its derivation is detailed and the associated computational strategies are discussed. Est1 is integrated within an adaptive greedy algorithm that is used to iteratively build the reduced-order model (ROM). The performance of Est1 is compared with the standard error estimator and a recently proposed one from [4]. It is shown that Est1 outperforms both existing estimators. Numerical experiments are performed on real-life microwave devices such as narrowband and wideband antennas, as well as a dual-mode waveguide filter. These examples show the capabilities and efficiency of the proposed methodology.

A modification of Est1, called Est2, is targeted specifically at electromagnetic systems [2]. Est2 makes use of the *Helmholtz decomposition* to significantly reduce the offline computational cost. A greedy ROM generation procedure is also proposed using Est2. The beneficial performance of this new error estimator for electromagnetic systems is illustrated through numerical experiments on real-life microwave devices and filters.

References

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