Parametric reduced order modelling for transport dominated systems via shifted POD deep learning models

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Model reduction for transport dominated phenomena using conventional linear ROM techniques suffers from a very low convergence rate, theoretically quantified by Kolmogorov n-width, which renders these methods practically infeasible for these class of problems. So far, several methods have emerged for such problems one such being shifted POD [4, 3] which was introduced to speed up the convergence. This method essentially decomposes transport fields by shifting the data field in a so-called co-moving frame, in which the travelling wave is stationary and can be described with very few spatial basis functions given by POD. Reduced order modelling for the parametrized PDEs rely on offline-online computational splitting. The expensive task of building the low dimensional subspace out of the FOM snapshots is performed once in the so called offline stage and the ROM approximation corresponding to any new parameter value is computed in the online stage. Besides the linear PDEs for which these methods work well, the problem arises when the dimension of the linear trial subspace becomes very large or when the hyper-reduction strategy scales with the dimension of the FOM. These are often recurrent issues when dealing with non-linear time dependent parametrized PDEs. As a parallel alternative, research on non-intrusive methods have picked up pace in recent years. These methods are purely data driven and feature an offline (training) phase and an online (testing/prediction) phase [1, 2]. Usually these methods employ deep learning frameworks to efficiently learn the nonlinear trial manifold corresponding to the training data and then predicts the solutions for unseen parameter values in the testing phase. In this talk, we address a possible way to extend such non-intrusive methods based on shifted POD. The idea is to learn the low dimensional dynamics created by the frames of the shifted POD using artificial neural networks. With the obtained description we are then able to predict states for unseen parameter values efficiently. The proposed method is tested on one- and two-dimensional examples, including combustion systems and incompressible flows around moving geometries to show the generality of the concept and the appropriate computational savings.

References

[1] S. Fresca, L. Dede, and A. Manzoni. A comprehensive deep learning-based approach to reduced order modeling of nonlinear time-dependent parametrized pdes. 2020.

[2] S. Fresca and A. Manzoni. Pod-dl-rom: Enhancing deep learning-based reduced order models for nonlinear parametrized pdes by proper orthogonal decomposition. *Computer Methods in Applied Mechanics and Engineering*, 388:114181, 2022.

[3] J. Reiss. Optimization-based modal decomposition for systems with multiple transports. *SIAM Journal on Scientific Computing*, 43(3):A2079–A2101, 2021.

[4] J. Reiss, P. Schulze, J. Sesterhenn, and V. Mehrmann. The shifted proper orthogonal decomposition: A mode decomposition for multiple transport phenomena. 2015.