# Dynamical low rank approximation and parametric reduced order models for shallow water moment equations

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Standard shallow water models assume a constant horizontal velocity over the water height. This nonphysical assumption is overcome by recently developed shallow water moment equations (SWME) [3, 4], which model a varying velocity profile using a potentially high-order polynomial. However, this leads to a large system of coupled PDEs for the evolution of the polynomial coefficients, even though the solution might evolve on a lower-order manifold.
In this work, we investigate two different approaches to reduce the complexity of the SWME. Firstly, we apply a dynamical low-rank approximation [2], which represents the solution as a low-rank factorization, e.g. with help of the Proper Orthogonal Decomposition (POD). The dynamical low-rank approximation framework provides time evolution equations for the individual factors, which can be solved with significantly reduced computational cost. Consequently, the method can be interpreted as a Galerkin method that updates basis functions according to the solution dynamics. Secondly, we introduce intrusive and non-intrusive parametric reduced order models [1] using POD-Galerkin and POD deep-learning methods. In this approach we set up the POD-basis a priori by computing snapshots of the SWME for different time-parameter instances. Predictions of the resulting Galerkin or deep-learning reduced order model are compared to the dynamical low-rank approximation concerning accuracy and runtime.

## References

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