

# Nonlinear manifold ROM with Convolutional Autoencoders and Reduced Over-Collocation method

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Non-affine parametric dependencies, nonlinearities and advection-dominated regimes of the model of interest can result in a slow Kolmogorov  $n$ -width decay, which precludes the realization of efficient reduced-order models based on linear subspace approximations. Among the possible solutions, there are purely data-driven methods that leverage autoencoders and their variants to learn a latent representation of the dynamical system, and then evolve it in time with another architecture. Despite their success in many applications where standard linear techniques fail, more has to be done to increase the interpretability of the results, especially outside the training range and not in regimes characterized by an abundance of data. Not to mention that none of the knowledge on the physics of the model is exploited during the predictive phase. In order to overcome these weaknesses, we implement the nonlinear manifold method introduced by Carlberg et al [2, 1] with hyper-reduction achieved through reduced over-collocation and teacher-student training of a reduced decoder [3]. A scheme of the new procedure is shown in Figure 1 <sup>1</sup>. We test the methodology on a 2d nonlinear conservation law and a 2d shallow water models, and compare the results obtained with a purely data-driven method for which the dynamics is evolved in time with a long-short term memory network.

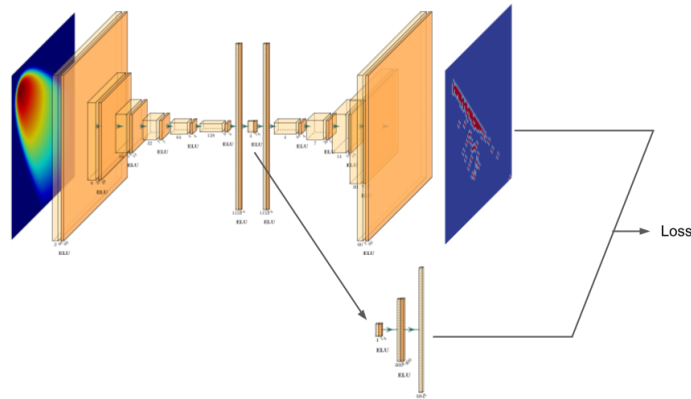


Figure 1: Teacher-student training of the compressed decoder for the 2d non-linear conservation law test case. The magic points, which the snapshots are restricted to, are shown in red over the domain.

## References

- [1] Y. Kim, Y. Choi, D. Widemann, and T. Zohdi. A fast and accurate physics-informed neural network reduced order model with shallow masked autoencoder. *arXiv preprint arXiv:2009.11990*, 2020.
- [2] K. Lee and K. T. Carlberg. Model reduction of dynamical systems on nonlinear manifolds using deep convolutional autoencoders. *Journal of Computational Physics*, 404:108973, 2020.
- [3] F. Romor, G. Stabile, and G. Rozza. Non-linear manifold rom with convolutional autoencoders and reduced over-collocation method. *arXiv preprint arXiv:2203.00360*, 2022.

<sup>1</sup>Made with the open-source package from GitHub <https://github.com/HarisIqbal88/PlotNeuralNet>.