# Multiscale modeling of heterogeneous structures based on a localized model order reduction approach

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Many of today's problems in engineering demand reliable and accurate prediction of failure mechanisms of mechanical structures. Thus, it is necessary to take into account the heterogeneous structure on the smaller scale, to capture the underlying physical phenomena. However, this poses a great challenge to the numerical solution since the computational cost is significantly increased by resolving the smaller scale in the model. Moreover, in applications where scale separation as the basis of classical homogenization schemes does not hold, the influence of the smaller scale on the larger scale has to be modelled directly.

This work aims to develop an efficient concurrent methodology to model heterogeneous structures combining the variational multiscale method (VMM) [4] and model order reduction techniques (e. g. [2]). First, the influence of the smaller scale on the larger scale can be taken into account following the additive split of the displacement field as in the VMM. Here, also a decomposition of the global domain into subdomains, each containing a fine grid discretization of the smaller scale, is introduced. Second, local reduced approximation spaces for the smaller scale solution are constructed by exploring possible solutions for each subdomain based on the concept of oversampling [3] and the solution of the associated transfer operator problem [1]. Herein, we propose to choose the training data based on the solution of a reduced global problem to incorporate the actual physical behaviour of the structure of interest and to extend it by random samples to ensure sufficient approximation capabilities in general. The local reduced spaces are designed such that local contributions of each subdomain can be coupled in a conforming way. Thus, the resulting global system is sparse and reduced in size compared to the direct numerical simulation, leading to a faster solution of the problem.

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