Dictionary-based Online-adaptive Structure-preserving Model Order Reduction for Parametric Hamiltonian Systems

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Reduced Basis (RB) methods applied to a non-linear time-dependent system often lead to a reduced basis, that is too large, such that no sufficient speed up can be realized compared to the high-order simulation. In general, the development of online-efficient RB methods for non-linear equations is still a strongly investigated topic. To address this problem, we present symplectic dictionary-based online-adaptive methods for Hamiltonian systems. The idea is, that the reduced basis is adapted during the time-stepping of the reduced simulation. Basis vectors, that are no longer required to represent the current solution vector (because, they would have a zero coefficient) are removed from the basis and new basis vectors are added, that are necessary to get a good approximation for the next time steps. We focus on the symplectic model order reduction of parametric Hamiltonian systems

$$\dot{x}(t) = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix} \nabla_x H(x(t), \mu) = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix} L(\mu)x(t) + \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix} f_{\mathrm{nl}}(x(t), \mu). \tag{1}$$

To compute a basis, a dictionary of snapshots $D_{\mathrm{X}} = \{x_1, ..., x_{N_{\mathrm{X}}}\}$ is constructed and during the online phase snapshots are selected from the dictionary. Then, the basis is computed from the selected snapshots. With an offline-online splitting this basis computation can be performed highly efficient, such that the run-time is just depending on the number of selected snapshots. Like this, we derive online-efficient versions of both classical basis generation techniques like POD as well as symplectic basis generation techniques, like the complex SVD-algorithm (see [1]). Furthermore, new symplectic methods like the Complex Gram–Schmidt method (also with online-efficient versions, see [2]) are developed.

In order to efficiently treat non-linearities in the right-hand side of (1) in combination with the SDEIM-algorithm see [3], a dictionary of non-linearity snapshots $D_{\mathrm{F}} = \{f_{\mathrm{nl}}(x_1), ..., f_{\mathrm{nl}}(x_{N_{\mathrm{F}}})\}$ and a dictionary of DEIM-indices $D_{\mathrm{P}} = \{i_1, ..., i_{N_{\mathrm{P}}}\}$ is computed. With an offline-online splitting an online-efficient version of the classical SDEIM-algorithm is obtained.

Different methods for selecting the snapshots from the dictionaries are developed and tested on a linear and a non-linear wave-equation model. The influence of the basis changes on the conservation of the Hamiltonian is studied.

References

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