Tensor Galerkin Proper Orthogonal Decomposition for Uncertainty Quantification of PDEs with Random Parameters

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The statistically sound treatment of modelled uncertainties in simulations comes with significant additional computational costs. Since a deterministic model can already be arbitrarily complex, running statistics for general problems may soon become infeasible unless some kind of model reduction is involved.
In this talk, we present a multidimensional Galerkin Proper Orthogonal Decomposition (POD) that simultaneously reduces the physical dimensions of the model and the dimensions related to the uncertainties; see [2] for details.
Using basic tensor calculus we extend our recent work of space-time Galerkin POD [1] to arbitrary dimensions and apply it to PDEs with multivariate uncertainties. By means of a numerical example we illustrate the procedure, how it outperforms POD based on random snapshots and how it compares to statistics informed greedy sampling strategies as proposed in [3].

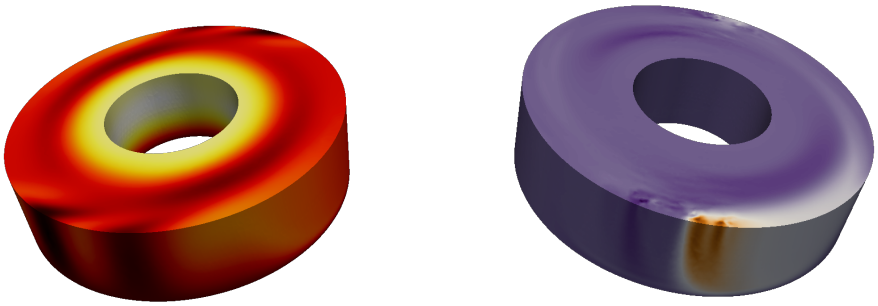

Figure 1: Illustration of the state of an example convection-diffusion simulation and the spatially distributed error in the expected value of the solution under a multivariate uncertainty in the diffusion parameter.

References

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[3] P. Chen, A. Quarteroni, and G. Rozza. A weighted reduced basis method for elliptic partial differential equations with random input data. *SIAM J. Numer. Anal.*, 51(6):3163–3185, 2013. doi:10.1137/130905253.