Impact of the Convergence of Series Expansions on Model Reduction of Quadratic-Bilinear Systems

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Quadratic-bilinear (QB) systems arise naturally from a large class of problems, including semi - discretizations of fluid flow systems governed by Burgers' equation or by the Navier-Stokes equations. Consequently, the model order reduction (MOR) of QB systems has been studied extensively in the literature [1, 3, 5]. Several MOR approaches for QB systems are based on so-called moment-matching, in which one expands the state in an infinite series, truncates the expansion after k (typically $k = 2$) terms, computes the generalized transfer functions of these terms, and computes a reduced order model (ROM) whose first k transfer functions interpolate the corresponding transfer functions of the full order model (FOM). While this approach has been successful in several cases, the impact of the truncation of the series on the quality of the resulting ROM has received little attention. The papers [2, 4] report that the ROMs computed using a straight forward application of the so-called Loewner framework for systems governed by Burgers' equation or the Navier-Stokes equations exhibit instabilities. This raises the question whether the convergence of the series, and its early truncation, contributes to these poor ROMs. In this talk, it is shown that the convergence of the series expansion indeed plays a critical role in the quality of the ROM. In particular, for systems governed by Burgers' equation, numerical experiments indicate that when the series expansion does not converge, the ROM is unstable, whereas when the series does converge, the resulting ROM is stable. Theoretical results on the convergence of the series expansion, which are tailored to systems governed by Burgers' equation and which expose the impact of viscosity, a critical parameter in Burgers' equation, are also provided.

References

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