Model reduction of descriptor systems with quadratic output functional

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This work is dedicated to the model reduction of differential-algebraic systems with quadratic output (`DAE_QO`) functional of the form

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + Bu(t), \qquad x(0) = 0, \\ y(t) &= x(t)^T M x(t). \end{aligned}$$

Under some mild conditions, these systems can be transformed into a Weierstraß canonical form and thus decouple into a differential equation and an algebraic equation, and the corresponding proper and improper states. In contrast to the case of differential-algebraic systems with linear output (`DAE_LO`), the system `DAE_QO` presents a coupling of the proper and improper states in the output signal. Our goal is to determine the dominant subspaces of the differential and algebraic states and then reduce the system accordingly.

To this end, we propose new Gramians that encode the suitable subspaces and demonstrate their relationship with energy functions. Firstly, we show that reachability is encoded by the proper and improper reachability Gramians, similarly as in the `DAE_LO` case. For the observability, we decompose the system output into four subsystems, two of those associated with proper states and the other two with improper states. Hence, based on this subsystem decomposition, we derive new observability Gramians for the proper and improper state vectors. They can again be determined by solving continuous-time and discrete-time Lyapunov equations. Finally, the proper reachability and observability Gramians are then used to derive a reduced differential state-space model using balanced truncation. Additionally, the improper Gramians are used to truncate uncontrollable and unobservable algebraic states. Furthermore, we derive an error estimator, which is used to evaluate the quality of the reduced surrogate model. Finally, we illustrate the effectiveness of our method by applying it to example problems.

References

[1] P. Benner, P. Goyal, and I. Pontes Duff. Gramians, energy functionals, and balanced truncation for linear dynamical systems with quadratic outputs. *IEEE Transactions on Automatic Control*, 67(2):886–893, 2022.