Hyper-reduction of geometrically parameterized nonlinear microstructures

Theron Guo[1], **Ondřej Rokoš**[2], **and Karen Veroy**[1]

[1]*Department of Mathematics and Computer Science, Eindhoven University of Technology*
[2]*Department of Mechanical Engineering, Eindhoven University of Technology*

To study the effect of microstructures onto effective macroscopic properties, two-scale simulations are often employed, where the macroscopic constitutive model is replaced by a microscopic partial differential equation (PDE) that is defined on a representative volume element (RVE), and which needs to be solved at every macroscopic quadrature point. Such simulations are typically expensive and therefore infeasible in multi-query contexts such as material design or optimization. To overcome this issue, the microscopic PDE needs to be replaced by a fast-to-evaluate surrogate model. Since the final goal is to employ the surrogate model for two-scale shape optimization, it must be accurate for a wide range of loading and geometrical parameters.

One popular method for parametric model order reduction is the Reduced Basis (RB) method [3], in which one finds a reduced solution space from pre-computed solution snapshots, for example using a proper orthogonal decomposition (POD). However, to achieve a high speed up for non-linear problems, a further reduction, termed hyper-reduction, is required. Hyper-reduction deals with finding a reduced set of integration points and corresponding weights. One possible approach is the Empirical Cubature Method (ECM) [1]. Geometrical parameters are typically dealt with by mapping the snapshots onto a parent domain. When the transformation map is known, the problem can then be solved on the parent domain. In the context of RB, related works often find these transformations using either free-form deformations or radial basis functions (see, e.g., [4]). However, these transformations are rather inflexible.

In this contribution, we obtain the transformation map by solving an auxiliary linear PDE (see, e.g., [2]). With the transformations available for all snapshots, we utilize POD to find a reduced basis on the parent domain, and, by using an adapted version of ECM, we then obtain a reduced set of integration points and weights that is accurate for a wide range of geometries.

To validate the methodology, several composite microstructures are tested, consisting of hyperelastic and elasto-plastic materials, and considering rotations and shape variations of the inclusions. At last, the surrogate model is tested in a full two-scale example, where, compared with the full simulation, a high accuracy in both local and global quantities as well as a high speed up is achieved.

References

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