Interpolatory (P)MOR via low-rank (tensor) approximation in general linear matrix equations

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It is a well known fact that interpolation of the transfer function

$$H(s) = C(sE - A)^{-1}B \tag{1}$$

in certain points $s_j = \mathrm{i}\omega_j \in \mathrm{i}\mathbb{R}$, in projection-based model order reduction, can be achieved via projection to rational Krylov subspaces [1]. Further it has been shown that the solutions X and Y of the Sylvester equations

$$-AX + EXS = BL, \tag{2}$$

$$-A^{\mathsf{T}}Y + E^{\mathsf{T}}YS = C^{\mathsf{T}}L, \tag{3}$$

with the spectrum of S equal to the set of all s_j, span these exact Krylov subspaces, as long as none of the s_j is a pole of H and (S, L) is controllable. It is a well observed fact that the low rank of the right hand sides often transfers to a low (numerical) rank of the solutions X and Y. This is especially true when many interpolation points are used, i.e. S gets large. Our solvers aim to exploit this fact to automatically decide about the reduced orders after massive oversampling.
When H additionally depends on parameters, i.e. there are parameter dependencies in E, A, B, or C, we derive tensor versions of (2), (3) and suggest low-rank tensor solvers to compute the truncation matrices.

References

[1] A. C. Antoulas, C. A. Beattie, and S. Gugercin. *Interpolatory Methods for Model Reduction.* Computational Science & Engineering. Society for Industrial and Applied Mathematics, Philadelphia, PA, 2020.