Effective A-posteriori Error Estimation for Port-Hamiltonian Systems

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The modeling of modern technical systems typically includes the consideration of different physical domains to represent a realistic behavior. The method of port-Hamiltonian (pH) systems manages to describe such systems in a unified framework by defining the energy as the *lingua franca* between the involved subsystems [1]. High-dimensional systems result from the spatial discretization of these models, leading to computationally demanding simulations and making model order reduction (MOR) techniques indispensable to allow for real-time scenarios or multi-query simulations.

Typically structure-preserving MOR techniques are used to preserve the pH structure and with it some of its advantages, such as passivity or modularity. The control of the introduced approximation error requires the usage of an error estimator. Error estimation approaches help to build confidence in the reduced model and can be used to work on adaptive basis generation, e.g. greedy algorithms, to improve the projection basis iteratively.

Based on the knowledge of different (non-)structure-preserving basis generation and projection techniques from our previous work [2, 3], the current work aims at developing an a-posteriori error estimator based on residuals for port-Hamiltonian systems by taking advantage of the specific pH-structure. One problem of current mathematical error estimators like [4] is the overestimation for large realistic systems retained from commercial FE which makes these unsuitable for instance in uncertainty quantification (UQ) in parameter identification. This problem is eased by introducing a so-called auxiliary linear problem (ALP), which provides an approximation for the residual, that helps to increase the effectivity of the error bound [5]. The obtained results are illustrated by a sophisticated example from the field of fluid-structure interaction.

References

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