Model order reduction for wave-type problems with band-limited outputs of interest

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In this talk, we present a goal-oriented model order reduction (MOR) technique targeting wave-type problems. Constructing reduced order models for wave-type problems poses various challenges, such as, potentially slow Kolmogorov n-width decay [1] in contrast to exponential decay for certain linear elliptic coercive problems, the inability to preserve properties of the original model, and potentially weak stability properties. We propose constructing reduced order models in the frequency domain, where the parametric partial differential equation is an elliptic problem. In many applications the output of interest is band-limited. Therefore, the input-to-output map for these applications is amenable to reduction. Inspired by the time domain POD-greedy algorithm [2], we propose a POD-Greedy algorithm driven by a goal-oriented error estimator in the frequency domain to generate reduced basis functions. In each greedy iteration, we apply a POD-based compression technique to iteratively enrich the basis and benefit from the band-limited output of interest to guarantee rapid convergence of the reduced basis approximation. Moreover, targeting the basis generation to the considered output further helps to obtain a reduced order model of small dimension. The reduced basis functions preserve the structure of the full-scale problem during projection and generate stable reduced order models. In addition, the resulting reduced order models do not jeopardize the time domain accuracy after numerical Laplace inversion of the frequency domain approximation. The proposed algorithms are widely applicable to various seismological applications with a seismogram as output of interest, and to structural health monitoring applications with damping. Numerical experiments for a 2D seismic wave equation with semi-realistic data functions demonstrate an exponentially converging reduced basis approximation and stable reduced order models.

References

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