# Symplectic Model Reduction of Hamiltonian Systems on Nonlinear Manifolds

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Classical model reduction techniques project the governing equations onto linear subspaces of the high-dimensional state-space. However, for problems with slowly decaying Kolmogorov-n-widths such as certain transport-dominated problems [6], classical linear-subspace reduced order models (ROMs) of low dimension might yield inaccurate results. Thus, the reduced space needs to be extended to more general nonlinear manifolds. Moreover, as we are dealing with Hamiltonian systems, it is crucial that the underlying symplectic structure is preserved in the reduced model, see [5, 7].

To the best of our knowledge, existing literatures addresses either model reduction on manifolds, see e.g. [4], or symplectic model reduction for Hamiltonian systems, but not their combination. In this talk, we bridge the two aforementioned approaches by providing a novel projection technique called *symplectic manifold Galerkin*, which projects the Hamiltonian system onto a nonlinear symplectic trial manifold such that the reduced model is again a Hamiltonian system, see [1]. We derive analytical results such as stability, energy-preservation and a rigorous a-posteriori error bound. Moreover, we construct a weakly symplectic convolutional autoencoder in order to computationally approximate the nonlinear symplectic trial manifold. We numerically demonstrate the ability of the method to outperform structure-preserving linear subspace ROMs results for a linear wave equation for which a slow decay of the Kolmogorov-n-width can be observed, see e.g. [2, 3].

## References

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