Deep Orthogonal Decomposition via Mesh-Informed Neural Networks for Reduced Order Modeling of parametrized PDEs

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In the context of parametrized PDEs, Reduced Order Models (ROMs) allow for an efficient approximation of the parameter-to-solution map, which is extremely useful whenever dealing with expensive many-query routines such as constrained optimization, sensitivity analysis and uncertainty quantification. Recently, motivated by the limitations of classical approaches such as the Reduced Basis method, many authors have been considering the use of Deep Learning techniques for building non-intrusive ROMs, e.g. [1, 3, 5, 4]. Within this framework, we propose a novel approach, Deep Orthogonal Decomposition (DOD), where a Deep Neural Network (DNN) yields a representation of the solution manifold in terms of an adaptive local basis. In principle, due to the very high-dimensions involved, designing and training such DNN models can be a challenging task. As a remedy, we exploit Mesh-Informed Neural Networks (MINNs), a novel class of architectures that was recently introduced in [2]. MINNs embed their hidden layers into discrete functional spaces of increasing complexity, obtained through a sequence of meshes defined over the underlying PDE domain. This results in sparse models that are computationally less demanding and thus better suited for implementing the DOD. We assess the robustness of the proposed approach by running different numerical experiments, including domains that feature an involved geometrical shape, and high-dimensional parameter spaces. The method is also compared with other state-of-the-art approaches, such as those involving Principal Orthogonal Decomposition and Autoencoders.

References

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