Analysis of Hyper Reduction for the Computation of Nonlinear Normal Modes

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Nonlinear Normal Modes(NNMs) are defined as the invariant manifolds that correspond to the Lyapunov subcenter manifolds for nonlinear structural dynamic systems. In case of conservative systems, an NNM reduces to a periodic solution of the system and is the extension of the concept of linear eigenmodes for nonlinear systems. The variety of nonlinear phenomena e.g. bifurcation, chaos and internal resonances poses a challenge in the setup of reduced order models (ROMs)[2].

The Galerkin projection of the system into a significantly smaller subspace is commonly employed in Model Order Reduction (MOR) of nonlinear Finite Element(FE) models [4]. As a consequence of nonlinear phenomena, the a-priori selection of adequate basis vectors presents a difficulty. Additionally, a computational bottleneck is created in the computation of the nonlinear force vector as it needs to be evaluated in the full model space.

To remove this bottleneck, hyper reduction methods are used to compute the nonlinear forces in the reduced subspace. The Energy Conserving Sampling and Weighting (ECSW) method selects a limited subset of elements and assigns them a weighting factor to reduce the computational effort of the nonlinear force evaluation by then only evaluating that limited subset [1]. A direct computation of the nonlinear force in the reduced subspace is enabled by the Stiffness Evaluation Procedure (STEP) that sets up a reduced stiffness tensor [3].

This work consists of the computation of NNMs for a doubly clamped beam by using the Multi Harmonic Balance method in combination with a continuation algorithm. Different reduction bases are created by using modal derivatives and linear eigenmodes. The major novelty is the application of HR with ECSW and STEP to the computation of NNMs. A qualitative analysis of the NNMs of the reduced system in comparison to the full system is presented. Furthermore, the computational effort needed to compute the NNMs is analyzed for different ROMs.

References

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