Adaptive Localized Reduced Basis Methods in Multiscale PDE-Constrained Parameter Optimization

T. Keil[1], L. Mechelli[2], S. Rave[1], F. Schindler[1], M. Ohlberger[1], and S. Volkwein[2]

[1]*Mathematics Münster, University of Münster*
[2]*Department of Mathematics and Statistics, University of Konstanz*

PDE-constrained parameter optimization problems with large- or multiscale applications easily exceed computational resources if standard approximation methods are employed for the underlying forward problem. For the involved parameterized partial differential equations, the reduced basis (RB) method is a model order reduction (MOR) method allowing for an efficient and certified approximation of the solution manifold. Admittedly, for problems with a slow convergence of the Kolmogorov n-width, e.g., with a high-dimensional parameter space, the offline phase for constructing a sufficiently accurate surrogate model can be prohibitively large.
To remedy this, we discuss more recent approaches that go beyond the classical offline-online splitting of MOR methods and adaptively build a surrogate model along the optimization path. The design of error-aware trust-region reduced basis (TR-RB) methods [1, 4] allows for localizing the reduction with respect to the parameter space and enables a certified optimization method with a high convergence rate. On the other hand, localized model order reduction techniques can replace the high-dimensional full-order model in scenarios where standard approximation schemes fail. A recently proposed approach in the context of multiscale problems is the two-scale reduced basis localized orthogonal decomposition method (TSRBLOD) [3], which is particularly suitable for the design of localized TR-RB methods [2]. In addition, we can use a relaxation of the outer trust-region optimization loop, also proposed in [2], allowing for a rigorous convergence result but converges much faster due to larger step sizes in the initial phase of the iterative algorithm.
In this talk, we present many different aspects of the TR-(L)RB algorithm with particular emphasis on a posteriori error estimation, the convergence of the overall optimization method, and numerical experiments that demonstrate the applicability of our approach. To this end, we focus on several different aspects of the algorithm in both the optimization loop as well as the full- and reduced-order models.

References

[1] T. Keil, L. Mechelli, M. Ohlberger, F. Schindler, and S. Volkwein. A non-conforming dual approach for adaptive trust-region reduced basis approximation of pde-constrained parameter optimization. *ESAIM. Mathematical Modelling and Numerical Analysis*, 55(3):1239, 2021.

[2] T. Keil and M. Ohlberger. A relaxed localized trust-region reduced basis approach for optimization of multiscale problems. *arXiv preprint arXiv:2203.09964*, 2022.

[3] T. Keil and S. Rave. An online efficient two-scale reduced basis approach for the localized orthogonal decomposition. *arXiv preprint arXiv:2111.08643*, 2021.

[4] E. Qian, M. Grepl, K. Veroy, and K. Willcox. A certified trust region reduced basis approach to PDE-constrained optimization. *SIAM Journal on Scientific Computing*, 39(5):S434–S460, 2017.