# Mass-conserving and energy-consistent ROMs for the incompressible Navier-Stokes equations with time-dependent boundary conditions

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In the reduced order modeling of the incompressible Navier-Stokes equations, significant interest is paid to the preservation of fundamental physical structures such as the mass conservation and the energy balance. However, many existing methods are limited to fluid flows with homogeneous or inhomogeneous, but time-independent boundary conditions [3, 4]. In this work, we propose a mass-conserving and energy-consistent reduced order model (ROM) for fluid flows with time-dependent boundary conditions.

We consider an energy-conserving finite volume discretization on a staggered grid [4] and decompose the velocity field into a component satisfying homogeneous boundary conditions and a time-dependent lifting function. Other works involving such lifting functions require that the boundary conditions are parametrized by a small number of time-dependent coefficients [2]. On the contrary, we allow arbitrary boundary conditions and apply Proper Orthogonal Decomposition to snapshots of the boundary conditions to obtain an approximation of the boundary conditions. The resulting ROM satisfies the mass conservation equation exactly with respect to these approximated boundary conditions. Moreover, the ROM is velocity-only, i. e. the velocity field can be computed without knowledge of the pressure field. Thereby, we avoid additional computational costs and pressure-related inf-sup-instabilities. Furthermore, due to our choice of the lifting function, which is inspired by the Helmholtz-Hodge decomposition [1], the ROM satisfies an kinetic energy evolution which matches corresponding evolutions of the full order model and the continuous model. As a consequence, the ROM is observed to be more accurate and stable compared to ROMs which violate this kinetic energy evolution.

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