# Deep-Reinforcement-Learning-informed Adaptive Refinement for High-order Discontinuous Galerkin Methods

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Discontinuous Galerkin (DG) finite element methods have enjoyed considerable success as a flexible and robust technique for the numerical solution of partial differential equations [3]. One of the advantageous features of DG methods is its element-locality and weak element-to-element coupling, which allow for straightforward adaptive refinement, given an error estimator. The discontinuous nature of the DG polynomial spaces in which the numerical solution is sought additionally provide a natural error estimator, the so-called non-conformity (NCF) indicator [5]. The NCF estimator is based on the assumption that the exact solution is physically continuous, implying that jumps in the numerical solution can be seen as a measure of error. However, despite the generality of this estimator, it is unable to take into account temporal or non-local patterns and remains a relatively uninformed heuristic. On the other hand, deep reinforcement learning provides a very general framework for learning action policies in complicated settings by rewarding good strategies and penalizing undesirable ones through trial and error [1]. Recent work to incorporate deep learning into finite element methods is in its infancy, and has focused on either on only classical continuous Galerkin schemes or on other aspects of DG-FEM unrelated to adaptive refinement [6, 2, 4]. We investigate the application of deep reinforcement learning as an approach to augment or replace the state-of-the-art error estimators such as the NCF and heuristic adaptive refinement strategies. We demonstrate the methodology on test cases in computational physics.

## References

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