Subspace-Distance-Enabled Active Learning for Parametric Model Order Reduction of Dynamical Systems

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In scenarios where repeated evaluations for varying parameter configuration of a high-fidelity physical model are required, surrogate modeling techniques based on model order reduction are used. The reduced basis method [1], equipped with a residual-based error estimator, is one possible methodological choice. There, snapshots at new parameter samples are generated and the reduced basis space is greedily updated via an iterative procedure. However, when the governing equations of the dynamics are not accessible, we need to construct the parametric reduced-order model (ROM) in a non-intrusive fashion. In this setting, the residual-based error estimate for optimal parameter sampling is not available. Our work intends to provide an alternative optimality criterion to efficiently populate the parameter snapshots, thereby, enabling us to efficiently construct a parametric ROM.

In contrast to the reduced basis method, we consider separate parameter-specific proper orthogonal decomposition (POD) subspaces. We design several metrics based on the subspace distance which can be used to extract the most important parameter values. One approach is to first generate low-accuracy (on a coarse grid for fast computation) snapshots for a preliminary fine parameter set. Then using the metrics, we pick the most important parameter locations for which high-accuracy snapshots are generated. An alternate approach is to first start with high-accuracy snapshots for a preliminary coarse parameter set. We then sample new parameter locations by progressively subdividing the farthest subspaces, in an iterative fashion, based on the metrics. We also present heuristically sound strategies to perform this subdivision between any two selected subspaces. Ultimately, a parametric ROM based on kernel-based shallow neural networks is actively learned with the proposed framework.

To demonstrate the validity of our proposed ideas, we present numerical experiments using several physical models. For instance, consider the parametric viscous Burgers' equation. Figure 1 shows the first 35 parameter selections and their corresponding POD dimension when using the subspace-distance-enabled active learning (SDE-ActLearn) procedure. And figure 2 shows the performance of the ROM at out-of-training time instances t for an out-of-training viscosity value of 10^{-3}.

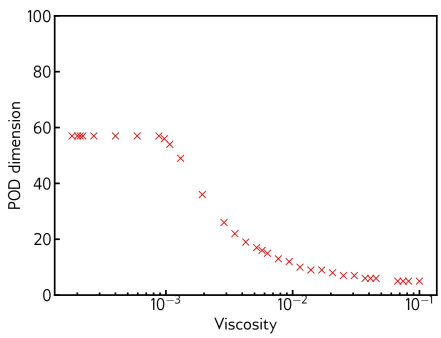

Figure 1: Selected parameters and corresponding POD subspace dimension for Burgers' equation.

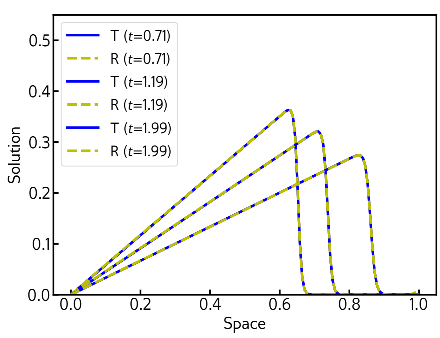

Figure 2: Comparison of ROM (R) and true (T) solutions for Burgers' equation with viscosity 10^{-3}.

References

[1] J. S. Hesthaven, G. Rozza, and B. Stamm. *Certified Reduced Basis Methods for Parametrized Partial Differential Equations*, volume 590. Springer, 2016.