

Data-driven learning of coarse basis functions in adaptive FETI-DP

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The convergence rate of FETI-DP domain decomposition methods is generally determined by the eigenvalues of the preconditioned system. For second-order elliptic partial differential equations, coefficient discontinuities with a large contrast can lead to a deterioration of the convergence rate. A remedy can be obtained by enhancing the coarse space with elements, which are often called constraints, that are computed by solving small eigenvalue problems on portions of the interface of the domain decomposition, i.e., edges in two dimensions or faces and edges in three dimensions. This approach is denoted as adaptive FETI-DP. For many elliptic partial differential equations, this leads to a provably robust method.

In general, it is difficult to predict where these constraints have to be computed, i.e., on which edges or faces. Using a machine learning strategy based on neural networks the geometric location of these edges or faces can be predicted in a preprocessing step. This reduces the number of eigenvalue problems that have to be solved. Numerical experiments for model problems and realistic microsections using regular decompositions as well as those from graph partitioners are provided, showing very promising results.

In a further step, using deep learning, we directly learn the constraints and build a surrogate model which replaces the eigenvectors associated with the local eigenvalue problems. Numerical results show the robustness of this approach.