Long-time prediction of nonlinear parametrized dynamical systems by deep learning-based reduced order models

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Conventional reduced order models (ROMs) anchored to the assumption of modal linear superimposition, such as proper orthogonal decomposition (POD), may reveal inefficient when dealing with nonlinear time-dependent parametrized PDEs, especially for problems featuring coherent structures propagating over time. To enhance ROM efficiency, we propose a nonlinear approach to set ROMs by exploiting deep learning (DL) algorithms, such as convolutional neural networks. In the resulting DL-ROM [2, 5], both the nonlinear trial manifold and the nonlinear reduced dynamics are learned in a non-intrusive way by relying on DL algorithms trained on a set of full order model (FOM) snapshots, obtained for different parameter values. Performing then a former dimensionality reduction on FOM snapshots through POD enables, when dealing with large-scale FOMs, to speedup training times, and decrease the network complexity, substantially [4].
A further step has led us to introduce LSTM neural networks instead of convolutional autoencoders, thus obtaining the μt-POD-LSTM-ROM technique that better grasps the time evolution of the PDE system [1]. This framework allows us to perform extrapolation of the PDE solution forward in time, that is, on a (much) larger time domain than the one used to train the neural network, for unseen values of the input parameters - a task often missed by traditional projection-based ROMs. Accuracy and efficiency of the resulting μt-POD-LSTM-ROM are assessed on several examples, ranging from low-dimensional, nonperiodic systems to applications in structural mechanics dealing with MEMS [3], obtaining faster than real-time simulations that are able to preserve a remarkable accuracy.

References

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