

# High Order Approximations of the Operator Lyapunov Equation Have Low Rank

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We present a low-rank greedily adapted  $hp$ -finite element algorithm for computing an approximation to the solution of the Lyapunov operator equation. We present a model for analyzing the regularity in eigenfunctions of the solution of the Lyapunov equation which we utilize to justify the use of high order finite element spaces. We also present an a posteriori error estimator and show its effectiveness on the model problem. We use a spectral theoretic setting in order to define the error estimator as well as to define measures of the approximation error. On the example of the Laplace operator on the dumbbell domain we achieve eight figures of accuracy for computing the trace of the solution of the Lyapunov equation using a finite element space of dimension of only  $10^4$  degrees of freedom. Even more surprising is the observation that  $hp$ -refinement has an effect of reducing the rank of the approximation of the solution.