# Nonlinear model order reduction for hyperbolic conservation laws by means of diffeomorphic transformations of space-time domains

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Parametrized hyperbolic conservation laws constitute a severe difficulty for model order reduction due to moving and interacting shock fronts and discontinuities. The solution manifold suffers from a slow decay of the Kolmogorov $n$-width, see [1] and [4], making it impossible for linear approaches like the reduced basis method to achieve accurate approximations.
We propose a nonlinear approach based on diffeomorphic transformations of space-time domains, allowing to align shocks, and thus, to overcome this drawback of linear methods. By considering solutions depending on space and time, shock interactions are directly incorporated into the snapshots. The transformations are treated as elements from the diffeomorphism group, which forms a Lie group, i.e. it has the structure of a smooth manifold. Associated with the Lie group of diffeomorphisms is the Lie algebra of smooth vector fields that parametrizes parts of the Lie group by geodesic evolution in the diffeomorphism group. The Lie algebra of smooth vector fields forms a vector space. Therefore, we can apply well-known algorithms from linear model order reduction to obtain a reduced model for the vector fields, see for instance [5]. Numerical experiments show the potential of the proposed algorithm for hyperbolic problems like Burgers' equation.
The method uses ideas similar to geodesic shooting in image registration, mainly developed in [2], and shows strong connection to differential geometry and the theory of Lie groups and Lie algebras. Further, the approach is motivated by the method of freezing, discussed in [3], which was restricted to problems without interaction of discontinuities.

## References

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