# Solving parametric PDEs with an enhanced model reduction method based on Linear/Ridge expansions

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Classical projection-based model reduction methods, like the reduced basis method [1, 2], are popular tools for getting quickly solvable reduced order models for parametric PDEs. Unfortunately, for some problems the error-decay with respect to the dimension of the projection space is predetermined to be slow, e.g., for transport or wave equations with jump discontinuities [2]. Therefore nonlinear methods are needed that do not project onto a linear space.
Our approach is to build the reduced order model with linear basis functions that are enhanced with some ridge functions [3, 4]. A ridge function consists of a one dimensional profile function as well as a direction that expands the profile to the multivariate domain. For the difficult task of finding the optimal directions, we developed a particle grid algorithm. The Linear/Ridge function space consisting of linear basis functions as well as linear combinations of ridge functions with parameter-dependent directions, is clearly nonlinear.
Offline, we first build the linear basis with known techniques by using snapshots that are solutions of the parametric PDE. If the error does not decay fast enough by adding more basis functions, we enhance the linear space with ridge functions. This is done by iteratively picking a worst parameter, computing the associated snapshot and determining directions with the particle grid algorithm for the profiles to approximate the snapshot at best. Then, extending the ridge function space by adding new profiles to the basis set until the approximation is precise enough for this specific snapshot. This procedure is repeaded until the overall error is sufficient small. Finally with the generated Linear/Ridge function space, the parametric PDE can be solved online-efficient by using interpolation for the directions [3].

## References

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