Structured barycentric forms and their application to iterative data-driven model reduction of second-order systems

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Data-driven and reduced-order modeling are essential tools for computing high-fidelity compact dynamical models that approximate real-world physical phenomena. Here, data are represented by frequency response measurements, i.e., samples of the transfer function of the unknown model. There are many methods available to tackle this problem in the unstructured standard case; see, e.g., [1]. In particular, we mention here the vector fitting (VF) algorithm from [2] and the AAA algorithm [3], which both iteratively fit transfer functions to data by means of solving linear(ized) least-squares problems.
The key ingredient of these data-driven approaches is the *barycentric form* of the system's rational transfer function. This is an advantageous representation of general rational functions, that:

- provides an easy transition between rational functions and the matrix representation of (linear time-invariant) systems;
- directly imposes interpolation conditions at selected support points.

In this work, we present extensions of the classical barycentric form to the case of mechanical systems described by second-order differential equations:

$$\begin{aligned} M\ddot{x}(t) + D\dot{x}(t) + Kx(t) &= B_{\mathrm{u}}u(t), \\ y(t) &= C_{\mathrm{p}}x(t), \end{aligned} \tag{1}$$

with the transfer function $H(s) = C_{\mathrm{p}}(s^2M + sD + K)^{-1}B_{\mathrm{u}}$. In particular, we make use of these barycentric forms to develop various new structure-preserving reduced-order modeling approaches to obtain systems of the form (1) from given frequency domain data. We present the following approaches:

- an extension of the VF algorithm for fitting modally damped mechanical systems (1) in [4];
- extensions of the AAA for (1) by imposing one-sided interpolation conditions, and by fitting other conditions in a least-squares manner.

For the latter, we show that by carefully choosing the interpolation points (in a greedy manner, inspired by classical AAA), structured reduced models (1) can be constructed to enforce reliable approximations.

References

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